Alternative Thinking 2024 | Issue 4

Can Machines Build Better Stock Portfolios?

The Virtue of Complexity in the Cross-Section of Stocks

Executive Summary

In the second issue of our 2024 Alternative Thinking series, we showed that machine learning techniques can be used to help improve market timing strategies.[1] We now extend these concepts to constructing stock selection strategies following a similar framework.

The relation between predictor variables (i.e., signals) and stock returns is a complicated, unknown, and complex function. Recovering it from simple linear approximations is likely to be very limited.

One way machine learning techniques can be used to help improve stock selection strategies is by picking up nonlinearities between the predictor variables and returns in the cross-section. Given the unknown nature of nonlinearities, more "complex" models—those with a large number of predictor variables, which may exceed the number of observations—have greater efficacy.

More complex models can better identify true nonlinear relationships and, thus, produce better stock selection strategy performance. This "virtue of complexity" result is validated in practical multi-factor stock selection applications in which long/short optimal portfolios are formed using three signal sets: value and momentum, Fama-French 5-factor model plus momentum, and a suite of defensive-oriented signals. Our results indicate performance improvements relative to a simple, linear approach in the range of 50-100%, suggesting that machine learning can help to build better stock selection portfolios.

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1 Alternative Thinking 2024 Issue 2.

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About the Portfolio Solutions Group

The Portfolio Solutions Group (PSG) provides thought leadership to the broader investment community and custom analyses to help AQR clients achieve better portfolio outcomes.

We thank Alfie Brixton, Jeff Cao, Pete Hecht, and Bryan Kelly for their work on this paper. We also thank Jordan Brooks, Jorge Fernandez-Cuervo, Antti Ilmanen, Thom Maloney, Toby Moskowitz, and Dan Villalon for helpful comments.

Introduction

Finance naturally has low predictability and small numbers of time series observations,[2] suggesting small, simple models—not complex machine learning models—are best suited for investment applications.[3] However, new research is challenging this principle of parsimony.[4] The Alternative Thinking 2024 Issue 2 showed that expected returns are likely nonlinear in the underlying predictor variables (i.e., signals), and small, simple market timing models miss this important relationship. Large, complex models are able to pick up the nonlinearities and produce better market timing performance. In fact, more complex models perform better out-of-sample—a so-called "virtue of complexity." The virtue of complexity holds for timing traditional markets, such as stocks and bonds, and timing long/short factors, such as value, momentum, and low beta; see **Exhibit 1** for US equities timing strategy performance.[5]

Exhibit 1: Hypothetical Out-of-Sample Equity Market Timing Performance Using Well-Documented Predictors

January 1, 1927 – December 31, 2020

We use 15 predictor variables from Goyal-Welch (2008): default-yield spread, inflation, stock variance, dividend payout ratio, long-term gov bond yield, term spread, t-bill rate, pure credit return, dividend-price ratio, dividend yield, long-term gov bond return, earnings-price ratio, book-to-price, net equity expansion, and lagged market return.

	US Stocks Excess Return	Simple Linear Timing Strategy Return	Complex Nonlinear Timing Strategy Return
Sharpe Ratio	0.51	-0.12	**0.47**
Appraisal Ratio		-0.19	**0.31**
Alpha t-Stat		-1.74	**2.88**
Skew	-0.41	-1.29	**2.54**

Source: AQR. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=517667. Results are gross of transaction costs. The above complex results use T=12, P=12,000 and the variables do not have lookahead bias. The dependent variable is the excess return of the S&P 500 index. The statistics reported are for static exposure to the U.S. stock market return, the simple timing strategy, and the nonlinear timing strategy. Please read the disclosures in the Appendix for a description of the investment universe and the allocation methodology used to construct the Hypothetical Simple and Complex U.S. Equity Timed Market Returns backtest. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix hereto. No representation is being made that any investment will achieve performance similar to those shown. For illustrative purposes only and not representative of a portfolio AQR currently manages.

While the previous paper focused on a time series, market timing application, the virtue of complexity also holds cross-sectionally when forming stock selection portfolios. For example, consider the following two popular stock selection factors: value and momentum. What's the highest Sharpe ratio portfolio that can be constructed with these two factors, and can we materially improve upon this result by constructing additional factors which are nonlinear combinations of the original two signals? The answer to the latter question is yes—a cross-sectional variation of the virtue of complexity.[6]

2 See Israel, Kelly, Moskowitz (2020).
3 See Asness, Ilmanen, Maloney (2015).
4 See Kelly, Malamud, and Zhou (2021).
5 Please refer to the Alternative Thinking 2024 Issue 2.
6 See Didisheim et al. (2023, 2024).

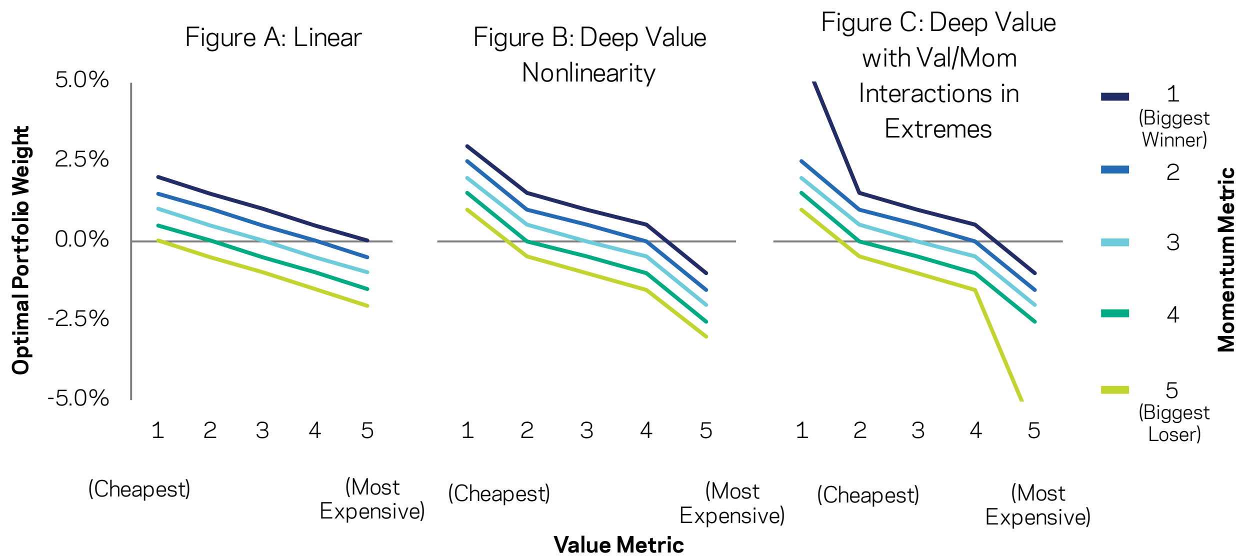
What Are Some Possible Nonlinearities in Expected Returns and Optimal Portfolio Weights?

Let's start with a concrete example demonstrating how a simple linear portfolio rule may differ from a more complex nonlinear one.

Assume a stock's optimal portfolio weight is driven by two signals, valuations (i.e., value) and relative short-term performance (i.e., momentum). When valuations are low (i.e., cheap) and short-term relative performance has been good, the stock's optimal portfolio weight is positive. In contrast, when valuations are high and short-term relative performance is poor, the stock's optimal portfolio weight is negative. **Exhibit 2 Figure A** depicts this relationship in a linear manner. But what if there is an additional expected return impact when valuations are at extremes—so called "deep value"?[7] This would impact the optimal portfolio weight in a nonlinear manner as highlighted in **Exhibit 2 Figure B**. Additional nonlinearities between the signals and returns can be incorporated, such as having an outsized impact on return (and, thus, optimal portfolio weight) when extreme valuations and momentum are aligned, i.e., when the stock is extremely cheap with unusually high relative short-term performance (**Exhibit 2 Figure C**).

Exhibit 2: Nonlinearity Examples in Expected Stock Return

Hypothetical Optimal Portfolio Weight of an Example Stock



Source: AQR. Market returns, "value metric," and "momentum metric" data are hypothetical and for illustrative purposes only. The value metric is a generic example metric of valuation, and the momentum metric is a generic metric of short-term relative performance. The x-axis plots the level of the value metric, the y-axis plots the optimal portfolio weight, and the 5 different lines indicate differing levels of the momentum metric as shown in the key on the right. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix. Not representative of any portfolio that AQR currently manages.

7 See Asness et al (2021).

Why Do More Complex Models Perform Better?

Complex models better reflect reality by picking up nonlinearities between the signals (X) and returns (R) in the cross-section (**Exhibit 3**). In other words, the optimal, Sharpe ratio-maximizing, portfolio weight (w) is a nonlinear function of the signals (X), and the complex model captures this relationship. In practice, the nonlinear relationships are unknown and must be estimated. This can be done by estimating a large, complex linear model where P new predictor variables (S) are generated by taking nonlinear transformations of the original signals (X).[8] Since the nonlinear predictors (S) can be interpreted as long/short portfolio weights,[9] F represents the return on a long/short factor based on the nonlinear predictors S. The optimal portfolio is approximated by a linear combination of the long/short nonlinear factors (F).

Exhibit 3: A Complex Multi-Factor Stock Selection Model

True Model: $\boldsymbol{w(X_t)'R_{t+1}}$

Empirical Model: $\boldsymbol{\lambda' S_t' R_{t+1} = \lambda' F_{t+1}}$

where:

$w(X_t)$ are the weights of the highest Sharpe Ratio portfolio and R are stock returns

S are nonlinear functions of X, the original predictors

F are long short factors based on the nonlinear predictors, S

λ represents the weights on the nonlinear long short factors, F

How many nonlinear transformations of the original signals should be used in the empirical model, i.e. should P be small or large? If we first focus on the portfolio expected return objective, a more complex model with higher P would better approximate the true, nonlinear portfolio weight function. As a result, more complex models with higher P deliver higher expected returns. However, complex models—models with few data points (T) and many parameters (P)—could be very difficult to estimate, increasing the volatility of out-of-sample portfolio returns. This is where regularization techniques, such as ridge, help out. As model complexity increases, the regularization techniques of ridge are able to identify a set of lambdas (λ) that fit the data and can be estimated with high precision (i.e., small variance). As a result, the portfolio Sharpe ratio increases with model complexity—the so-called virtue of complexity applied to stock selection portfolios. For a more detailed discussion on this topic, refer to the 2024 Alternative Thinking Issue 2.

8 How should one create these nonlinear predictors S, i.e., what is an appropriate nonlinear transformation of X? One option is random Fourier features: $S_i=[\sin(\gamma w_i'X),\cos(\gamma w_i'X)]'$ where $w_i \sim \text{i.i.d.}\,N(0,1)$ are random, normally-distributed weights applied to the original signals X, and γ is a scaling factor. The practitioner can generate as many nonlinear, random Fourier features as they want to best approximate and capture complex nonlinearities. It's important to note there are many other ways to generate nonlinear predictors. We chose random Fourier features because it is a convenient tool for illustrating the broad virtue of complexity principle.

9 One simple method to create long/short factor portfolio weights from any given predictor signal is as follows: Assume your signal is book-to-price (value). Rank the stocks based on book-to-price. Divide the rank by the number of stocks in the cross-section. Subtract the mean. This leads to a set of portfolio weights between -0.5 (short) and 0.5 (long), which can be scaled to any dollar long/short level the investor wishes.

Testing the Theory in Three Multi-Factor Contexts

So far, we have been focusing on the theory behind why more complex return models should deliver better stock selection strategy performance. Let's now apply the theory to three practical examples: forming long/short optimal portfolios using value and momentum, Fama-French signals plus momentum, and a suite of defensive-oriented signals.[10] Can complex models build better multi-factor stock selection portfolios? Yes.[11]

Value and Momentum

Our value and momentum US stock selection model uses book-to-price and trailing 12 month return for the raw signals,[12] with data from January 1, 1963 to December 31, 2019.[13] In order to identify nonlinear relationships between the signals and portfolio weights, we estimate a 360-month rolling covariance matrix and expected return vector with P = 2 to 36,000 long/short nonlinear factors.[14, 15] The predictor variables underlying the long/short nonlinear factors are generated by taking nonlinear combinations of the raw signals, book-to-price and last 12 months returns. The out-of-sample "complex" portfolio weights (i.e. lambdas) are estimated using standard Markowitz portfolio results.[16]

We also construct a "simple" model which uses only two long/short factors based off the original two raw signals.

The out-of-sample multi-factor stock selection performance using our complex model is reported in **Exhibit 4**. The complex strategy generates a 2.1 Sharpe ratio using a complexity of 100 (i.e. 36,000 nonlinear factors divided by 360 time series observations), which is materially higher than the simple model's 1.3. The correlation between the complex and simple (linear) returns is fairly modest at 0.5. As a result, the complex model is highly additive on top of the more simple linear approach (and, given the level of diversification, this would hold true even if the complex model yielded a Sharpe ratio commensurate with the simple one).

10 The Fama-French signals include value, size, investment, and profitability. The defensive-oriented signals include earnings volatility, profitability, financial leverage, and low beta.

11 Note that the results reported in this paper do not reflect the impact of transaction costs. Other AQR research has validated the virtue of complexity principle in an environment with transaction costs and other market frictions. To learn more about machine learning and transaction costs, please refer to "Machine Learning and the Implementable Efficient Frontier," Jensen, Kelly, Malamud, Pedersen (2022).

12 The trailing 12 month return excludes the most recent month.

13 The 1963 to 2019 time period studied is consistent with Didisheim et al (2023), which is the underlying academic paper motivating the analysis. The spirit of the results does not change using different timeframes.

14 This corresponds to a complexity (c) ranging from about 0.0056 to 100, where complexity is defined as the number of predictor variables (P) divided by the number of time series observations (T).

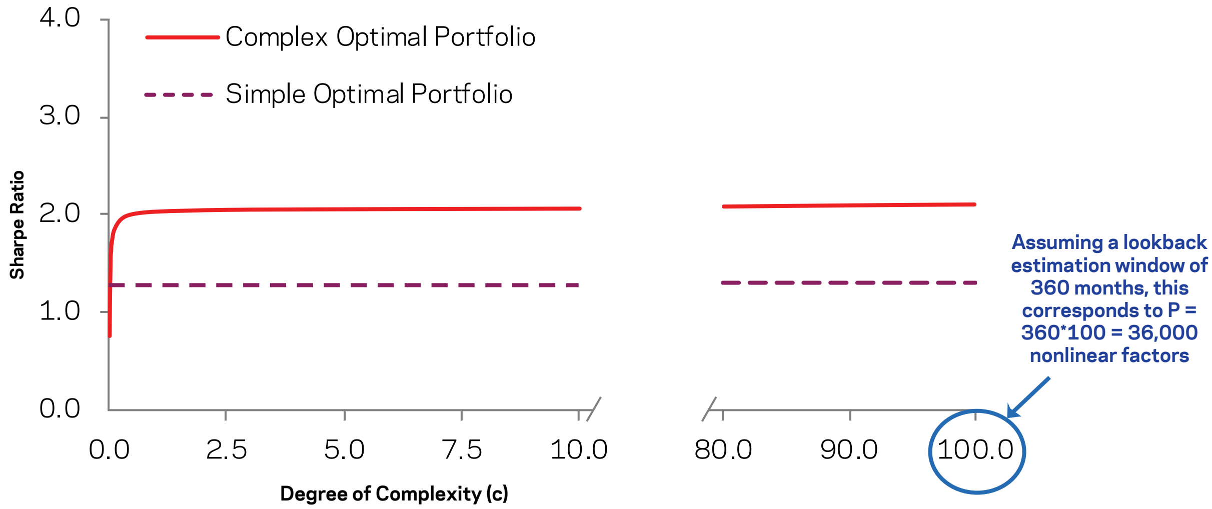
15 It's important to note that both the simple and complex portfolio models use a low frequency, monthly rebalances and b) slow moving, 30-year, historical covariance and expected return inputs. These design choices should partially mitigate the impact of transaction costs. Furthermore, the focus of the analysis is on the relative performance improvement from incorporating nonlinearities. While transaction costs degrade absolute performance measures, it's not clear how transaction costs impact relative performance measures. Transaction costs could increase with complexity, but we leave this topic for future research.

16 We estimate a rolling 360-month ridge regression using a shrinkage parameter (z) of 1. Ridge regression solves for the betas that minimize the following objective function: $\sum_{t=1}^{T}(y_t - \sum_{i=1}^{P} x_{t,i}\beta_i)^2 + z\sum_{i=1}^{P}\beta_i^2$. The z parameter penalizes large betas, shrinking the betas towards zero. The dependent variable (y_t) is a vector of ones. It can be shown that the regression betas represent the Sharpe Ratio-maximizing portfolio weights (λ). The choice of a 360-month look-back period is consistent with Didisheim et al (2023, 2024). Refer to Didisheim et al (2023, 2024) for additional robustness analysis.

Exhibit 4: Hypothetical Sharpe Ratio of Nonlinear Value and Momentum Model

January 1, 1963 – December 31, 2019

We use the Fama-French value and momentum raw characteristics to generate nonlinear features.



Source: AQR, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4388526. The Sharpe ratios are based on the Hypothetical Simple and Complex Nonlinear Value and Momentum Models. The red line uses a shrinkage parameter z = 1. The x-axis shows complexity (c = P / T) with P ranging from 1 to 36,000 and T = 360 months. In each subsample, the test assets are a set of P factor portfolios managed on the basis of nonlinear random Fourier features derived from the Fama-French value and momentum characteristics. Please read the disclosures in the Appendix for a description of the investment universe and the allocation methodology used to construct the Hypothetical Simple and Complex Nonlinear Value and Momentum Models backtest. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix hereto. No representation is being made that any investment will achieve performance similar to those shown. For illustrative purposes only and not representative of a portfolio AQR currently manages.

Fama-French and Momentum

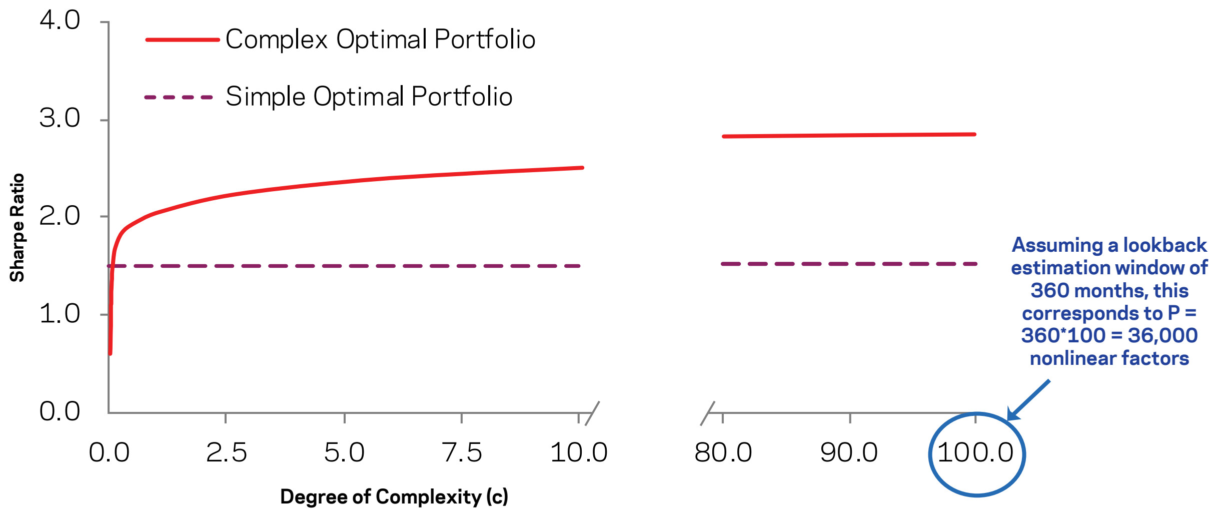
We repeat the exercise from the previous section with one change: in addition to book-to-price (value) and last 12 months return (momentum), we include three Fama-French-motivated raw signals: market equity (size), operating profit-to-book equity (profitability), and asset growth (investment). The out-of-sample multi-factor performance using our complex model is reported in **Exhibit 5**. Consistent with the previous results, the strategy generates a Sharpe ratio of 2.9 at a complexity of 100 (i.e. 36,000 nonlinear factors), which is approximately double that of the simple linear model.[17] Importantly, the complex model produces a diversifying return stream, and therefore is highly additive, relative to the simple model.

17 Repeating this exercise using only liquid, large cap stocks leads to a 65% Sharpe ratio improvement.

Exhibit 5: Hypothetical Sharpe Ratio of Nonlinear Fama-French 5-Factor + Momentum Model

January 1, 1963 – December 31, 2019

We use the underlying characteristics of the Fama-French 5-Factor model—excluding excess market return and including momentum: value, size, investment, profitability, and momentum to generate nonlinear features.



Source: AQR, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4388526. The Sharpe ratios are based on the Hypothetical Simple and Complex Nonlinear Fama-French 5-Factor + Momentum Models. The red line uses a shrinkage parameter z = 1. The x-axis shows complexity (c = P / T) with P ranging from 2 to 36,000 and T = 360 months. In each subsample, the test assets are a set of P factor portfolios managed on the basis of nonlinear random Fourier features derived from the five characteristics underlying the Fama-French 5-factor model, including momentum and excluding excess market return: size, momentum, value, investment, and profitability. Please read the disclosures in the Appendix for a description of the investment universe and the allocation methodology used to construct the Hypothetical Simple and Complex Nonlinear Fama-French 5-Factor + Momentum Models backtest. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix hereto. No representation is being made that any investment will achieve performance similar to those shown. For illustrative purposes only and not representative of a portfolio AQR currently manages.

Defensive-Oriented Signals

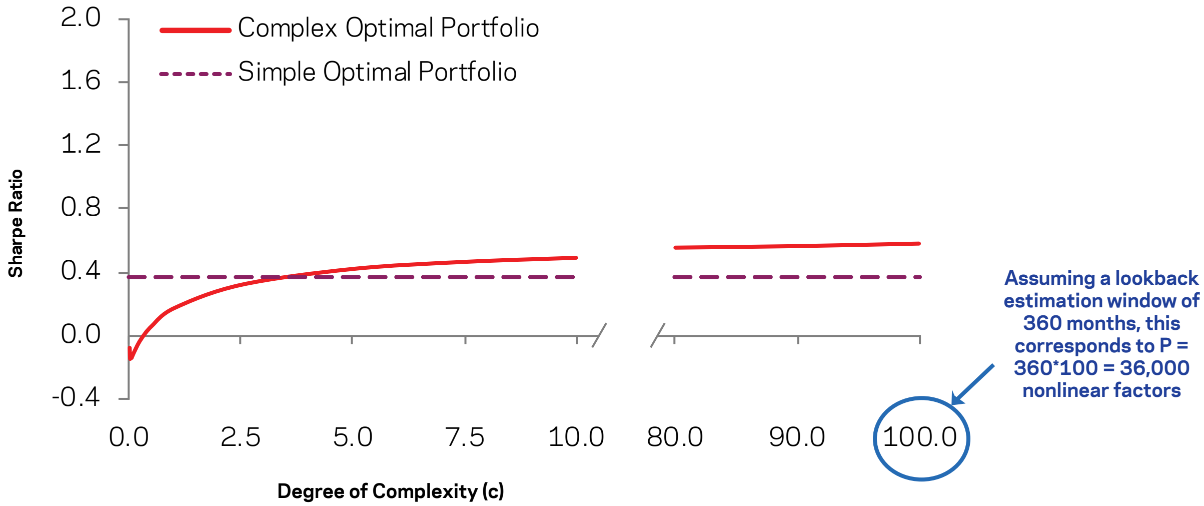
Lastly, we consider a more defensive-oriented signal set: earnings volatility, profitability, financial leverage, and low beta. While the level of the Sharpe ratios are lower across the board, the spirit of the results in **Exhibit 6** are similar to those found earlier.[18] The "complex" multi-factor strategy generates a 0.6 Sharpe ratio at a complexity of 100 (i.e. 36,000 nonlinear factors), which is approximately 60% higher than the "simple" strategy. Again, the complex model produces a diversifying return stream, and therefore is highly additive, relative to the simple model.

18 The level of Sharpe Ratios is lower because the long/short portfolio formation process imposes dollar neutrality, instead of market neutrality. When the signal set includes defensive characteristics such as low beta, this could lead to long/short defensive-oriented portfolios with negative market beta—which could drag down performance. This would not be a problem if the market portfolio were a traded factor in our optimization, but the market portfolio is excluded.

Exhibit 6: Hypothetical Sharpe Ratio of Nonlinear Defensive-Oriented Model

January 1, 1963 – December 31, 2019

For this run, we use defensive signals, including earnings volatility, profitability, financial leverage, and low beta.



Source: AQR, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4388526. The Sharpe ratios are based on the Hypothetical Simple and Complex Nonlinear Building a Better Defensive Portfolio Models. The red line uses a shrinkage parameter z = 1. The x-axis shows complexity (c = P / T) with P ranging from 2 to 36,000 and T = 360 months. In each subsample, the test assets are a set of P factor portfolios managed on the basis of nonlinear random Fourier features derived from the "Building a Better Defensive Portfolio Model": earnings volatility, profitability, financial leverage, and low beta. Please read the disclosures in the Appendix for a description of the investment universe and the allocation methodology used to construct the Hypothetical Simple and Complex Nonlinear Building a Better Defensive Portfolio Models backtest. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix hereto. No representation is being made that any investment will achieve performance similar to those shown. For illustrative purposes only and not representative of a portfolio AQR currently manages.

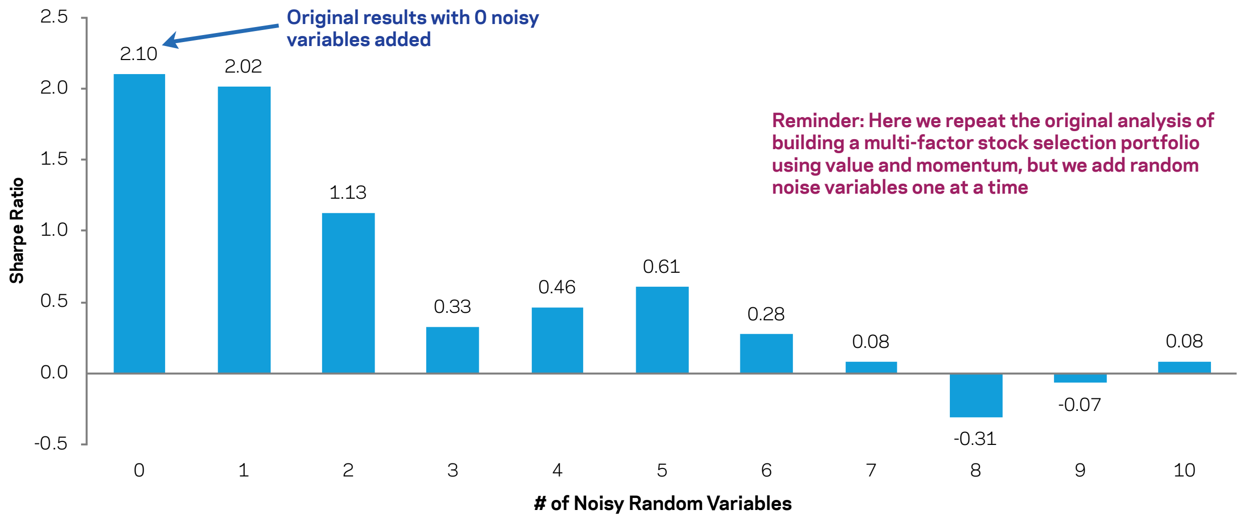
The Virtue of Complexity Is Not a License for Data Mining

Large, complex models can help build better stock selection portfolios.[19] However, similar to the market timing problem, this virtue of complexity is not a license for throwing any predictor variable into the portfolio optimization. It is critical that the underlying raw signals be related to the true nonlinear expected return and, thus, the true nonlinear portfolio weight. Including signals with no relationship to the true expected return degrades performance, as seen in **Exhibit 7.** If we use the earlier two signal "value and momentum" model, we achieve an out-of-sample Sharpe ratio of 2.1. As we include noisy predictor variables alongside the original two signals, strategy performance declines rapidly. Including just two noisy predictor variables reduces performance by approximately 50%.[20]

19 See Kelly et al (2022).
20 Due to sampling variation, the results do not monotonically decline as more noisy predictor variables are included. The reported Sharpe ratios are an average across 20 runs.

Exhibit 7: Hypothetical Out-of-Sample Nonlinear Value and Momentum Model Performance adding Noise Variables

January 1, 1963 – December 31, 2019



Source: AQR, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4388526. The Sharpe ratios are based on the Hypothetical Complex Nonlinear Value and Momentum Model. We use a shrinkage parameter z = 1. We use P = 36,000 and T = 360 months. In each subsample, the test assets are a set of P factor portfolios managed on the basis of nonlinear random Fourier features derived from the Fama-French value and momentum characteristics and 1 to 10 random normal variables, centered at zero with a standard deviation of 10. Please read the disclosures in the Appendix for a description of the investment universe and the allocation methodology used to construct the Hypothetical Simple and Complex Nonlinear Value and Momentum Models backtest. Hypothetical data has certain inherent limitations, some of which are disclosed in the Appendix hereto. No representation is being made that any investment will achieve performance similar to those shown. For illustrative purposes only and not representative of a portfolio AQR currently manages.

Concluding Thoughts

Using small, simple, linear models to build stock selection portfolios misses nonlinear relationships between the predictor variables and returns, leaving money on the table. Large, complex models overcome this limitation, better estimate the nonlinear relationship between predictors and optimal portfolio weights, and generate better stock selection performance—the so-called virtue of complexity in the cross-section. This virtue of complexity principle is validated in three stock selection applications: forming optimal portfolios using value and momentum signals, Fama-French model and momentum signals, and a suite of defensive-oriented signals. The out-of-sample “complex model” performance improvements range from 50 to 100%. Thus, the potential performance improvements from implementing complex models are meaningful.

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The **S&P 500 Index** is the Standard & Poor's composite index of 500 stocks, a widely recognized, unmanaged index of common stock prices.

The **Goyal and Welch (2008) variables** are from "A Comprehensive Look at The Empirical Performance of Equity Premium Prediction" (Goyal and Welch, 2008). Please refer to Goyal and Welch (2008) for detailed descriptions of the variables.

HYPOTHETICAL PERFORMANCE RESULTS HAVE MANY INHERENT LIMITATIONS, SOME OF WHICH, BUT NOT ALL, ARE DESCRIBED HEREIN. NO REPRESENTATION IS BEING MADE THAT ANY FUND OR ACCOUNT WILL OR IS LIKELY TO ACHIEVE PROFITS OR LOSSES SIMILAR TO THOSE SHOWN HEREIN. IN FACT, THERE ARE FREQUENTLY SHARP DIFFERENCES BETWEEN HYPOTHETICAL PERFORMANCE RESULTS AND THE ACTUAL RESULTS SUBSEQUENTLY REALIZED BY ANY PARTICULAR TRADING PROGRAM. ONE OF THE LIMITATIONS OF HYPOTHETICAL PERFORMANCE RESULTS IS THAT THEY ARE GENERALLY PREPARED WITH THE BENEFIT OF HINDSIGHT. IN ADDITION, HYPOTHETICAL TRADING DOES NOT INVOLVE FINANCIAL RISK, AND NO HYPOTHETICAL TRADING RECORD CAN COMPLETELY ACCOUNT FOR THE IMPACT OF FINANCIAL RISK IN ACTUAL TRADING. FOR EXAMPLE, THE ABILITY TO WITHSTAND LOSSES OR TO ADHERE TO A PARTICULAR TRADING PROGRAM IN SPITE OF TRADING LOSSES ARE MATERIAL POINTS THAT CAN ADVERSELY AFFECT ACTUAL TRADING RESULTS. THERE ARE NUMEROUS OTHER FACTORS RELATED TO THE MARKETS IN GENERAL OR TO THE IMPLEMENTATION OF ANY SPECIFIC TRADING PROGRAM, WHICH CANNOT BE FULLY ACCOUNTED FOR IN THE PREPARATION OF HYPOTHETICAL PERFORMANCE RESULTS, ALL OF WHICH CAN ADVERSELY AFFECT ACTUAL TRADING RESULTS. The hypothetical performance results contained herein represent the application of the quantitative models as currently in effect on the date first written above, and there can be no assurance that the models will remain the same in the future or that an application of the current models in the future will produce similar results because the relevant market and economic conditions that prevailed during the hypothetical performance period will not necessarily recur. Discounting factors may be applied to reduce suspected anomalies.

This backtest's return, for this period, may vary depending on the date it is run. Hypothetical performance results are presented for illustrative purposes only. In addition, our transaction cost assumptions utilized in backtests, where noted, are based on AQR Capital Management LLC's, ("AQR's") historical realized transaction costs and market data. Certain of the assumptions have been made for modeling purposes and are unlikely to be realized. No representation or warranty is made as to the reasonableness of the assumptions made or that all assumptions used in achieving the returns have been stated or fully considered. Changes in the assumptions may have a material impact on the hypothetical returns presented. Actual advisory fees for products offering this strategy may vary.

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Hypothetical Simple and Complex U.S. Equity Timed Market Returns

The Hypothetical Simple and Complex U.S. Equity Timed Market Return series are based on two predictive regressions from January 1, 1927 to December 31, 2020. The simple model estimates the rolling market weight ("betas" or "pi") based on 15 macroeconomic variables from Goyal-Welch (2008) using a simple, 12-month rolling OLS regression. The complex model transforms the 15 Goyal-Welch factors into 12,000 random Fourier features and uses a rolling 12-month ridge regression (with shrinkage parameter z=1000, gamma/standard deviation of 2) to estimate market weight. The estimated betas from these regressions are weighted by the market return (S&P 500 Index) which forms the market-timing strategy returns. Returns are gross of fees and transaction costs. For more details on this methodology, please refer to "The Virtue of Complexity in Return Prediction", Kelly, Malamud, Zhou (2021).

Hypothetical Simple and Complex Nonlinear Value and Momentum Models

The Hypothetical Simple and Complex Nonlinear Value and Momentum Models are based on two portfolios formed using value and momentum signals. The investment universe is the CRSP NYSE-listed US Stock database. We use book-to-market equity and price momentum (t-12 to t-1) data from Jensen, Kelly, and Pedersen (Journal of Finance, 2023) to construct factors; both signals are lagged one month to prevent lookahead bias. The simple model constructs long/short value and momentum factors by ranking the signals in the cross-section of all stocks; standardizing them by dividing by the sum of ranks, subtracting 0.5, and dividing by the square root of the number of stocks in the cross-section per month; and finally computing the final factor return as a weighted sum of the standardized ranks and stock returns. The simple model weight in each factor is computed from the rolling 360-month tangency portfolio, which is estimated using simple OLS regression. The complex model transforms the two signals into P=2 up to 3,600 random Fourier features (for Fourier random weights, the standard deviation is uniform randomly selected from [0.5, 0.6, 0.7, 0.8, 0.9, 1.0]), which are used to form long/short factors by the same rank/standardization process. The complex model weight in each factor is estimated using a rolling 360-month ridge regression with shrinkage parameter z=1. Finally, the estimated simple and complex factor weights are used to form the simple and complex model portfolios. Returns are gross of fees and transaction costs. For more detailed methodology, please refer to "APT or 'AIPT'? The Surprising Dominance of Large Factor Models", Didisheim, Ke, Kelly, and Malamud (2023).

Hypothetical Simple and Complex Nonlinear Fama-French 5-Factor + Momentum Model

The Hypothetical Simple and Complex Nonlinear Fama-French 5-Factor + Momentum Models are based on two portfolios formed using Fama-French 5-Factor Model (excluding the market return) and momentum signals: size, value, investment, profitability, momentum. The investment universe is the CRSP NYSE-listed US Stock database. For the underlying signals, we use market equity, book-to-market equity, asset growth, operating profits-to-book equity, and price momentum (t-12 to t-1) data, respectively, from Jensen, Kelly, and Pedersen (Journal of Finance, 2023) to construct factors; all signals are lagged one month to prevent lookahead bias. The simple model constructs five long/short factors (size, value, investment, profitability, and momentum) by ranking the signals in the cross-section of all stocks; standardizing them by dividing by the sum of ranks, subtracting 0.5, and dividing by the square root of the number of stocks in the cross-section per month; and finally computing the final factor return as a weighted sum of the standardized ranks and stock returns. The simple model weight in each factor is computed from the rolling 360-month tangency portfolio, which is estimated using simple OLS regression. The complex model transforms the five signals into P=2 up to 3,600 random Fourier features (for Fourier random weights, the standard deviation is uniform randomly selected from [0.5, 0.6, 0.7, 0.8, 0.9, 1.0]), which are used to form long/short factors by the same rank/standardization process. The complex model weight in each factor is estimated using a rolling 360-month ridge regression with shrinkage parameter z=1. Finally, the estimated simple and complex factor weights are used to form the simple and complex model portfolios. Returns are gross of fees and transaction costs. For more detailed methodology, please refer to "APT or 'AIPT'? The Surprising Dominance of Large Factor Models", Didisheim, Ke, Kelly, and Malamud (2023).

Hypothetical Simple and Complex Nonlinear Building a Better Defensive Portfolio Models

The Hypothetical Simple and Complex Nonlinear Building a Better Defensive Portfolio Models are based on two portfolios formed using defensive signals: earnings volatility, profitability, financial leverage, and low beta. The investment universe is the CRSP NYSE-listed US Stock database. For the underlying signals, we use earnings volatility, operating profits-to-book equity, book leverage, and Frazzini-Pedersen market beta data, respectively, from Jensen, Kelly, and Pedersen (Journal of Finance, 2023) to construct factors; all signals are lagged one month to prevent lookahead bias. The simple model constructs four long/short factors (earnings volatility, profitability, financial leverage, and low beta) by ranking the signals in the cross-section of all stocks; standardizing them by dividing by the sum of ranks, subtracting 0.5, and dividing by the square root of the number of stocks in the cross-section per month; and finally computing the final factor return as a weighted sum of the standardized ranks and stock returns. The simple model weight in each factor is computed from the rolling 360-month tangency portfolio, which is estimated using simple OLS regression. The complex model transforms the four signals into P=2 up to 3,600 random Fourier features (for Fourier random weights, the standard deviation is uniform randomly selected from [0.5, 0.6, 0.7, 0.8, 0.9, 1.0]), which are used to form long/short factors by the same rank/standardization process. The complex model weight in each factor is estimated using a rolling 360-month ridge regression with shrinkage parameter z=1. Finally, the estimated simple and complex factor weights are used to form the simple and complex model portfolios. Returns are gross of fees and transaction costs. For more detailed methodology, please refer to

"APT or 'AIPT'? The Surprising Dominance of Large Factor Models", Didisheim, Ke, Kelly, and Malamud (2023).

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